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INDIVIDUAL
COEFFICIENTS IN A WETTED-WALL TOWER - II

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A THESIS

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INDIVIDUAL COEFFICIENTS IN A WETTED-WALL TOWER - II

ABSTRACT

Heat and mass transfer coefficients were determined for the vaporization of water into air in a wetted-wall column. The individual coefficients for the air film alone were calculated as well as the over-all coefficients of transfer from the water surface to the turbulent air stream. Methods are presented for establishing conditions at the interface between the air and water films and at the boundary between the air film and the turbulent air core. A comparison is made between two procedures for approximating these conditions. In one case an assumed relationship is made between the heat and mass transfer coefficients and, in the second case, an analogy is assumed between heat and mass transfer under the same flow conditions. The available evidence is insufficient to permit a firm comparison of the two procedures. In each method, conditions in the column were obtained by taking measurements across the tower at various horizontal planes. Data are also shown for points in the column where the flow of heat reverses direction.

INDIVIDUAL

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INTRODUCTION

The concept of films existing adjacent to the interface between phases has been of considerable value in understanding the mechanism of transfer across surface boundaries.[1] As the principal resistance to interphase transfer is found in these films, a knowledge of the boundary conditions and the individual film coefficients is of primary importance in predicting heat or mass transfer.[2] In the early stages of this concurrent investigation, Shepherd[3] developed a method for estimating interfacial conditions between the air and water films in a wetted-wall column. With boundary conditions established, the individual coefficients can then be calculated.

In the previous work, driving forces were obtained by determining point to point conditions within the tower. As reported by Shepherd,[3] in practically all investigations covered in the literature only terminal conditions were

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[1] W. G. Whitman, Chemical and Metallurgical Engineering, 29, (1923).

[2] T. K. Sherwood, "Absorption and Extraction," pp. 28, 61, (1937).

[3] G. R. Shepherd, "Individual Coefficients in a Wetted-Wall Tower," M. S. Thesis, Georgia Institute of Technology, pp. 2, 6-17, (1947).

measured. It was, therefore, considered advisable to continue the work on the point to point technique.

Heat and mass transfer coefficients were determined by Shepherd[4] at an air velocity of about 9 feet per second over a relatively narrow humidity and temperature range during a particular run, the range being restricted by equipment limitations. In the current investigation, a new installation was constructed to permit greater flexibility.

The specific objectives of the current work were: (1) to determine the individual heat and mass transfer coefficients at an air velocity of about 15 feet per second; (2) to investigate conditions at points where the heat flow reversed direction; (3) to further evaluate the method developed by Shepherd for estimating interfacial conditions.

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[4] G. R. Shepherd, Ibid, (1947).

GLOSSARY OF TERMS

A - area of surface, square feet.

C - specific heat at constant pressure, B.T.U. per pound.

F - correction factor for change in area and velocity across tower, dimensionless.

G - mass velocity of air, pounds per hour.

h - coefficient of heat transfer for air film, B.T.U. per hour per square foot per unit difference in temperature.

h' - coefficient of heat transfer for air film and turbulent core, B.T.U. per hour per square foot per unit difference in temperature.

h'' - coefficient of heat transfer between air film - water film interface and center of air stream, B.T.U. per hour per square foot per unit difference in temperature.

h_L - coefficient of heat transfer for water film, B.T.U. per hour per square foot per unit difference in temperature.

H - average absolute humidity of air stream, pounds of water vapor per pound of dry air.

H_i - humidity at air film - water film interface.

H_B - humidity at boundary of air film and turbulent air core.

H_y - humidity at distance y from air film - water film interface.

k - coefficient of mass transfer for air film, pounds per hour per square foot per unit difference in humidity.

k' - coefficient of mass transfer for air film and turbulent core, pounds per hour per square foot per unit difference in humidity.

k'' - coefficient of mass transfer between air film - water film interface and center of air stream, pounds per hour per square foot per unit difference in humidity.

L - length of wetted-wall section, feet.

GLOSSARY OF TERMS (Continued)

R_o - radius, distance from center of air stream to the interface between the air and water films, feet.

R - radius, distance from center of air stream, feet.

r - $\frac{u_B}{u_{ave}}$

s - humid heat, B.T.U. per pound of dry air.

T - average temperature of water film, degrees F.

t - average temperature of air stream at any plane in tower, degrees F.

t_B - temperature of air at boundary of air film and turbulent core, degrees F.

t_i - temperature of air film - water film interface at any plane, degrees F.

t_y - temperature at distance y from the air film - water film interface, degrees F.

u - linear velocity of air stream at distance R from center of air stream, feet per second.

u_{ave} - average linear velocity of air stream, feet per second.

u_B - air velocity at boundary of air film and turbulent core, feet per second.

W - water rate, pounds per hour.

y - distance from air film - water film interface, feet.

α - ratio $\frac{t_B - t_i}{t - t_i}$.

α' - $\frac{t_B - t_i}{t'' - t_i}$.

α'' - $\frac{t - t_i}{t'' - t_i}$.

GLOSSARY OF TERMS (Concluded)

β - ratio $\frac{H_B - H_i}{H - H_i}$.

β' - $\frac{H_B - H_i}{H'' - H_i}$.

β'' - $\frac{H - H_i}{H'' - H_i}$.

$\frac{C\mu}{k_o}$ - Prandtl number, dimensionless with consistent units: C = heat capacity at constant pressure, μ = viscosity, k_o = thermal conductivity.

$\frac{\mu}{pD}$ - Schmidt number, dimensionless with consistent units: μ = viscosity, p = density, D = diffusivity.

Subscripts 1,2,....15 refer to horizontal sampling planes within the tower.

EXPERIMENTAL APPARATUS

The basic equipment requirements for the wetted-wall installation were: (1) a column for contacting a film of water with an air stream under various conditions of flow; (2) regulating equipment for controlling the flow rate and temperature of the influent air and water; (3) a pressure tank to reduce the humidity of the inlet air to a relatively constant value; (4) measuring devices for determining the humidity and temperature at points within the column. Photographs of the installation are shown in Figures 8, 9, and 10. A detailed description of the apparatus follows:

Tower

The tower consisted of a glass column with an inside diameter of 2.50 inches, an outside diameter of 2.9 inches, and an over-all length of 96 inches. To permit sampling, the column was provided with fifteen 0.25 inch diameter holes spaced 6 inches apart from bottom to top, the lowest and highest points also being 6 inches from the ends of the column. The points were numbered consecutively from the base to the top of the column and all points were in a single vertical plane passing through the axis of the tower; however, the even and odd numbered points were placed 180° apart to reduce disturbance of the water layer. Thus, on a particular side of the column the sampling points were 12 inches apart.

The top of the glass tower was ground and polished to insure an even overflow of water. Water was supplied from an overflow pan equipped with a sieve type baffle which made it necessary for the water to pass through the baffle before flowing to the tower. No wave motion could be detected in the overflow section of the pan with this arrangement.

A calming section which was constructed from 2 inch aluminum pipe 60 inches in length extended approximately 2 inches into the bottom of the tower, the annular space between the column and calming section being about 0.1 inches. Water was collected from the annular space in a brass catch-pan which provided a liquid s eal of 0.5 inches at the base of the column.

The calming section, catch-pan, tower proper, and overflow pan were housed in a rectangular steel frame. Direct support for the tower was obtained by cementing the glass column to the overflow pan which was mounted on the top section of the steel frame. There was no direct support between the catch-pan and the column, however, to prevent horizontal movement of the column three small brass pins were placed in the bottom of the catch-pan. The catch-pan was attached directly to the calming section.

Although every effort was made to obtain a column which met rigid specification as regards to constancy of bore diameter and to vertical alignment, it was finally necessary to use a column with certain undesirable features in order to avoid excessive delay in the work. The column described in the above installation consisted of two 48-inch lengths of tubing welded together. As a result of this type of construction, the vertical alignment of the column was poor. In mounting, it was necessary to place the bottom of the column about 3 inches off the top center line to obtain an unbroken film of water. Also, the column diameter was reduced about 0.15 inches at the welded joint. The effect of these design inadequacies on the data is covered in the section on "Experimental Errors."

It should be mentioned that a total of four columns were ordered and received before the column just described was used, the first three columns being rejected because of very poor alignment characteristics, and/or welded joint construction. Although a better column could have been obtained in a shorter length, this was not considered advisable in view of the work and recommendations of Shepherd.[5] The shorter 44-inch column used in the previous investigation severely restricted the temperature and humidity range which could be covered in a particular run. It is understood that glass tubing meeting rigid bore specifications, $\pm$ 0.0002 inches, is now available in lengths up to about 16 feet.[6]

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[5] G. R. Shepherd, Ibid, p. 34, (1947).

[6] Phoenix Precision Instrument Company; Philadelphia, Pa.

Inlet Water

Water was supplied to the overflow pan at the top of the tower from a constant head tank through a calibrated rotameter. The effluent water from the base of the column was then collected in a fifty-gallon reservoir and recirculated to the overhead tank by a centrifugal pump.

Although flow rates could be controlled within a narrow range from about 30 to 600 pounds per hour, the maximum rate was limited to about 100 pounds per hour in this investigation. The water splashed over the temperature and,or, humidity measuring equipment at rates above 150 pounds per hour. This, of course, resulted in the entrainment of water droplets in the air stream. No entrainment could be detected at rates below 150 pounds per hour; glass was placed perpendicular to the air stream in these entrainment tests. No difficulty was encountered with the air blowing the film off the tower wall at water rates as low as 10 pounds per hour provided the tower was clean.

Heating was accomplished by two electrical pipe heaters, the second heater being located about 12 inches from the pan. Separate variacs were provided for manually regulating each heater; during operation the water temperature could be controlled within a range of 0.3 degrees F.

Inlet Air

Air was supplied from a two stage compressor with a maximum capacity of 160 pounds per hour. The compressor operated at a maximum pressure of 120 pounds per square inch gage on a differential of 10 pounds per square inch. A regulating valve, located upstream from the tower installation, reduced variations in air supply pressure by about 60 per cent. Manual regulation was required to maintain a relatively constant flow rate. In practice, the rate was controlled within 5 per cent.

Flow rates were measured by an accurately constructed orifice of 0.500 inches diameter placed in a line of 1.049 inches diameter according to the specifications of Ebaugh.[7] The downstream tap was located 0.60 pipe diameters from the orifice. The pressure drop across the orifice and the total pressure in the pipe were measured by mercury manometers, and the air temperature was determined through a suitably placed thermometer well. Flow rates calculated by the equations of Ebaugh[7] agreed within one per cent of checks with a positive displacement meter at low rates of flow.

Heating was accomplished by two electrical heaters. The first was an ordinary pipe heater which was manually controlled by a variac. This heater was used to warm the air to within 1 to 5 degrees of the desired temperature. Final heat-

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[7] N. C. Ebaugh, Engineering Thermodynamics, p. 115 (1937).

ing took place in the second unit consisting of a coil of nichrome wire located inside the 1-inch air supply line. This second heater was automatically controlled by a relay activated by a mercury thermoregulator located in the calming section. Also, a variac was provided to prevent the second coil from overheating when energized. Under operating conditions, the temperature could be controlled within limits of 0.5 degrees F. The thermoregulator actually operated on a temperature differential of 0.05 degrees F, however, lag effects and variations in air flow adversely affected control.

To insure proper mixing of the air after heating, the second heater was located about 15 feet from the base of the calming section; also, two 90 degree ells were in this section of 1 inch pipe.

Humidity control was obtained by maintaining a relative constant air supply pressure. Also, a small expansion tank located between the pressure regulating valve and the orifice installation served as a water trap. As volume requirements were large, no attempt was made to completely dry the air.

Temperature Measurements

All temperature measurements were made with copper-constantan thermocouples with very careful consideration being given to wire size and to thermocouple preparation. The diameter of the wire used in this investigation was 0.009

inches and small thermocouples of about 1/64 inches in diameter were prepared by fusing the copper-constantan wire in an electric arc. An ice bath insulated in a dewar flask was used as the reference temperature junction for the dew-point temperatures; however, to cover the air and water temperature range more accurately the transition temperature of decahydrate sodium sulfate was used for the latter measurements. A standard thermometer was placed in the dewar flask to provide a constant check on the junction temperature. All measurements were made through a self-balancing potentiometer which was connected to a voltage regulator. Each thermocouple was calibrated against a standard thermometer furnished with a calibration curve by the manufacturer. The thermometer was calibrated in divisions of 0.05 degrees F and could be estimated to 0.01 degrees F. A single calibration curve was obtained for the thermocouples used in measuring the air and water temperatures.

As the rate of change of the air temperature with respect to distance was usually greater in a horizontal plane, each temperature probe extended horizontally into the column and then vertically upward for a distance of 1.25 inches to reduce temperature errors resulting from conduction along the leads. The thermocouple leads were passed through the 1/16 inch glass probe with the thermocouple extending about 1/16 inch beyond the vertical opening. Each end of the probe was then sealed. A stopper was used to support the probe in the particular sample port under investigation. Eight probes were prepared for making temperature traverses across the tower at any one of the 15 sampling points; water temperatures were measured with the same thermocouples. Temperatures could be read within 0,2 degree F.

Humidity Measurements

Humidity measurements were made by the dew-point method. The apparatus consisted of a very thin stainless steel plate cemented between two glass chambers of about 1.0 and 1.5 inches diameter respectively. As a mixture of warm and cold water was used to control the temperature of the plate, the equipment was mounted vertically with the larger water chamber on top to prevent the accumulation of air bubbles at the plate surface. The temperature was measured by a thermocouple on the surface of the plate, plate and thermocouple being calibrated as a unit. Humidity determinations were made by drawing air through the lower chamber under a vacuum of 5 inches of water.[8] Dew formation on the underside of the plate was then observed in a mirror. When dew formed and evaporated at essentially constant plate temperature this was considered to be the dew-point. Results were reproducible within 0.5 degrees F. Dew-points were converted

[8]A pressure correction was made in calculating the humidity.

to humidities by the tables of Zimmerman[9] which were used throughout this work.

Air was sampled from points within the tower by a 1/16 inch diameter probe which was connected to the dew-point apparatus by Saran tubing. In order to measure humidities and temperatures at the same points, the vertical dimensions of the humidity and temperature probes were the same. The probe extended horizontally into the column and then vertically upward for a distance of 1.25 inches.

EXPERIMENTAL PROCEDURE

In preparation for a run, air and water flow rates were first adjusted to the desired values. The electrical heaters were then turned on to bring the air and water to the respective specified temperatures. In each case, the initial heating was accomplished with the larger unit, and the temperature was adjusted to within less than 5 degrees of the control value before the second heater was turned on. For the air, an effort was made to regulate the variacs to both heaters to such a point that the relay in the automatic control circuit operated continuously. Equilibrium was considered to have been reached when the terminal condition of air and water were constant for a period of thirty minutes. From

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[9] O. T. Zimmerman and I. Lavine, Industrial Research Services' Psychrometric Tables and Charts, (1945).

four to five hours were usually required to reach equilibrium. Temperature and humidity traverses were then made at planes 1, 3, 5, 7, 9, 11, 13 and 15 at distances of 1/16, 1/8, 1/4, 1/2, 3/4, 1, and 1-1/4 inches from the tower wall. At the conclusion of the run the air supply valve was closed and the air in the tower was allowed to saturate. An approximation of the convection and radiation losses was made by the temperature drop of the water under these conditions. Ten to sixteen hours were required for a complete run. Constant attention was required to maintain the flow rates and temperatures within the limits previously covered. This situation extended the time of the runs considerably and increased the difficulty in making both temperature and humidity measurements.

In the early runs an effort was made to reduce the time consumed in removing and placing the measuring equipment by determining the humidity at the even numbered points and the temperature at the odd numbered points. However, the temperature and humidity distribution in the column was found to be unsymmetrical at the various horizontal planes and the extrapolation required by the above procedure was not considered valid. As a result of this distribution, the traverses were carried past the center of the column on later runs and the temperature and humidity were measured at the same points. The difficulties arising from non-uniform distribution are covered further under the section on "Experimental Errors."

EXPERIMENTAL ERRORS

As previously mentioned, the vertical alignment of the column was poor. With this condition, the water film on the wall of the tower was non-uniformly distributed. The measured values of T, t, and H then were not necessarily the average values at a particular plane in the tower. To detect this error in the data, the calculated values of T, t, and H were plotted against the distance, L, from the bottom of the tower. A curve was then drawn through the points to establish a relationship between t and L, and H and L. Values of t and H from this curve were then used in estimating t_i and H_i. This plot is shown in Figure 3.

The estimation of errors in temperature measurements resulting from conduction along the thermocouple leads was also complicated by the unsymmetrical distribution in the column. An attempt was made to check the error in the probe temperature by lowering a vertically suspended thermocouple into the top of the tower. With this procedure, no significant difference could be detected between the probe and vertically suspended thermocouple temperatures. A check was then made by taking readings on each side of the column. At the same distance from the center of the column; the measured temperature should be higher on one side if conduction along the leads is appreciable and the temperature distribution is uniform. No correlation could be obtained by this procedure,[10] i.e., the test only confirmed the non-uniform distribution in most cases. For this reason, no temperature correction was applied.[11] The non-uniform distribution in the column probably constitutes the largest error in the experimental data.

METHODS OF CALCULATION

The procedure used in estimating boundary conditions is described, and, following this discussion, the methods employed in calculating the individual coefficients are presented. All symbols are defined under "Glossary of Terms."

Estimation of Boundary Conditions

The basic relationships which result from a material and energy balance combined with rate transfer equations on

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[10]As previously covered, the probes extended horizontally into the tower and then vertically upward. Evidently, the vertical section of the probe reduced errors resulting from conduction.

[11]Towards the end of the work, conditions in the tower were simulated by placing the horizontal section of the probe in a water bath at different temperatures and measuring the error in air temperature against a standard thermometer. Only preliminary observations were made and an insufficient amount of data was collected to warrant making temperature corrections. However, it was roughly estimated that the error in measurements at the bottom of the tower and at points near the interface was from 1 to 2 degrees F; the change in temperature was, of course, large at these points. A modification in the probe design which would permit a complete traverse of the tower would aid greatly in estimating errors resulting from conduction.

a differential section of an absorption tower of height dL are presented by Sherwood.[12] Then, in the reference thesis, Shepherd[13] developed expressions for the specific application of these relationships to point to point conditions within a wetted-wall column. As these fundamental deviations have been previously covered in detail, they will not be repeated; however, the procedure used by Shepherd[13] for estimating boundary conditions will be described briefly. The relationships employed were:

$$(H_B - H_i) = (t_B - t_i)\,\frac{h}{ks}\,\frac{dH}{dt} \qquad (1)$$

and

$$(H - H_i) = (t - t_i)\,\frac{h}{ks}\,\frac{dH}{dt}\,\frac{\alpha}{\beta} \qquad (2)$$

where

$$\alpha = \frac{t_B - t_i}{t - t_i} = \frac{\dfrac{r\,C\mu}{k_o}}{1 - r + r\,C\mu/k_o} \qquad (3)^{14}$$

and

$$\beta = \frac{H_B - H_i}{H - H_i} = \frac{\dfrac{r\mu}{pD}}{1 - r + \dfrac{r\mu}{pD}} \qquad (4)^{15}$$

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12 T. K. Sherwood and C. E. Reed, "Applied Mathematics In Chemical Engineering," 1st edition, p. 68-76, (1939).

13 G. R. Shepherd, Ibid, pp. 6-17, (1937).

14 A. H. Gibson, "The Mechanical Properties of Fluids," p. 178, London; Blackie and Son Limited, (1925).

15 A. P. Colburn, "Relation Between Mass Transfer and Fluid Friction," Industrial and Engineering Chemistry, 22, pp. 967-970, (1930).

It must be clearly understood that equations (1) and (2) refer to point conditions within the tower at a particular horizontal plane.

In equation (2), the values for t, H, and $\frac{dH}{dt}$ can be calculated directly from the experimental data, however, the ratio $\frac{h}{ks} \frac{\alpha}{\beta}$ must be estimated. The ratio $\frac{\alpha}{\beta}$ was determined from equations (3) and (4) and a constant value of 0.95 was assumed for $\frac{h}{ks}$. Thus, assuming equilibrium at the interface, equation (2) can now be solved for t_i and H_i. On a plot of H vs. t, equation (2), of course, is the equation of a straight line through the points t, H and t_i, H_i with a slope of $\frac{h}{ks} \frac{dH}{dt} \frac{\alpha}{\beta}$.

With t_i known, the boundary temperature t_B between the air film and turbulent core was then determined from equation (3). For the corresponding humidity, H_B, equation (4) was not used as this expression was not believed to be a good approximation of conditions existing in mass transfer.[16] Values of H_B were obtained directly from the experimental data by estimating the point at which the rate of change of the humidity with respect to the distance from the interface became constant.[17]

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[16] G. R. Shepherd, Ibid, pp. 30-31, (1947).

[17] Actually, once the point t_i,H_i is fixed, the point t_B,H_B can be determined from equation (1) if either t_B or H_B is known as a constant value of 0.95 has been assumed for $\frac{h}{ks}$.

There appears to be some danger in applying the above procedure. For example, Shepherd[18] obtained an average value of 0.935 for the ratio $\frac{h}{ks}$ which was in excellent agreement with the assumed value of 0.95; however, the average value for the corresponding over-all ratio, $\frac{h'}{k's}$, was about 0.60. It would not appear that the difference of approximately 0.3 is indicative of the experimental data. As reported by Shepherd,[18] the available evidence indicates the ratio $\frac{h'}{k's}$ ranges from about 0.95 to 1.15 over a considerable range of velocities. Also, in the current work it was noted that the point t_i, H_i was usually too high at high negative values of $\frac{dH}{dt}$, and too low at low negative values of $\frac{dH}{dt}$ if a constant value was assumed for $\frac{h'}{k's}$.[19] The temperature, t_i, can be estimated as the interfacial temperature must be lower than the water temperature when both the air and water are cooling.

For these reasons, an alternative method of approximation was developed to estimate boundary conditions. A description of this procedure follows:

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[18] G. R. Shepherd, Ibid, pp. 9-10, 30-36, (1947).

[19] This is essentially the same as holding $\frac{h}{ks}$ constant as the ratio $\frac{\alpha}{\beta}$ is also relatively constant.

Equations (1) and (2) may be expressed in terms of the over-all coefficients h' and k'. This equation then is:

$$(H - H_i) = (t - t_i) \frac{h'}{k's} \frac{dH}{dt} \qquad (5)$$

Also, a similar expression relating conditions at the interface and the center of the column is:

$$(H'' - H_i) = (t'' - t_i) \frac{h''}{k''s} \frac{dH}{dt} \qquad (6)$$

where
$$\frac{h''}{k''s} = \frac{\alpha''}{\beta''} \frac{h'}{k's} \quad ; \qquad (7)$$

$$\alpha'' = \frac{t - t_i}{t'' - t_i} \quad ; \qquad (8)$$

and
$$\beta'' = \frac{H - H_i}{H'' - H_i} \qquad (9)$$

If the ratio $\frac{\alpha''}{\beta''}$ were known, an exact solution could, of course, be obtained from equations (5) and (6) for t_i and H_i; no assumption being required for $\frac{h'}{k's}$. In the present work, an identical analogy was assumed between heat and mass transfer under the same flow conditions, and the ratio $\frac{h'}{k's}$ was assumed to be equal to $\frac{h''}{k''s}$. The point t_i, H_i was then located by the intersection of a straight line through the points t, H and t", H" with the equilibrium line. The correctness of this approximation, of course, diminishes as the ratio $\frac{\alpha''}{\beta''}$ deviates from a value of 1.

In some instances the relationship between t" and H" could not be determined accurately from the experimental data. When this was the case, equation (5) was solved for

t_i and H_i by assuming a value of 1.00 for $\frac{h'}{k's}$, this method being essentially the same as employed by Shepherd. The limitations to both procedures are covered further under "Discussion of Results."

With t_i and H_i fixed, t_B and H_B were then determined directly from the data by plotting $\frac{t_y - t_i}{t'' - t_i}$ and $\frac{H_y - H_i}{H'' - H_i}$ vs. $\frac{y}{R_o}$ on logarithmic paper. The point at which the slope of the curves became 1 was then estimated; this point was considered to represent boundary conditions.[20]

Calculation of Average Conditions

To determine the average temperature and humidity at a particular horizontal plane in the tower, the measurements made across the tower must be corrected for velocity distribution and the change in cross-sectional area. The correction factor, F, for an annular space of inside radius R_1 and outside radius R_2 is:

$$F_{12} = \frac{2\int_{R_1}^{R_2} u\,\phi\,RdR}{u_{ave}\;R_o^2} \qquad (10)^{21}$$

The average temperature at a particular horizontal plane in the tower then is:

$$t = F_{12} \times t_{12} + F_{23} \times t_{23}, \text{ etc.}$$

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[20] W. H. McAdams, "Heat Transmission" 2nd edition, pp. 102-104, 159, (1942).

[21] This expression corrects for changes in volume only. Strictly speaking, the correction should be on a weight basis, however, the change in density due to the change in air temperature across the tower is small.

The expression for the average humidity is identical.

An expression for the relationship u ∅ R was obtained from the work of Stanton[22] as reported by McAdams.[23] Although the velocity distribution covered in this reference actually applies to isothermal conditions, the approximation is believed to be good as the change in the temperature of the air through the tower is relatively small.

In the original plot of the temperature and humidity data vs. the distance from the wall, the distribution near the interface is not known.[24] However, as the temperature drop through the water film is not great, the point t_i and H_i can be estimated. In this work t_i was assumed to be 2 degrees F. below the water temperature except at points in the tower where the heat flow reversed direction.

The measured temperature of the water was assumed to be the average temperature, no attempt is made in this work to estimate the boundary temperature of the water film corresponding to the air film temperature t_B.

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[22] T. E. Stanton, D. Marshall, and C. N. Bryant, "On the Conditions at the Boundary of a Fluid in Turbulent Motion," Proceedings Royal Society, A 97 p. 413, (1920).

[23] W. H. McAdams, Ibid, p. 104, (1942).

[24] In Figure 1, the point at which the temperature becomes essentially linear with respect to distance is not clearly shown. Thus, t_{B1} is actually lower than t_{B13}. This condition is readily seen from a logarithmic plot as shown in Figure 5.

The surface velocity and thickness of the water layer were calculated from the equations of Friedman.[25] These data apply to a static air core, however, Gilliland[26] has shown that air velocity has a negligible effect on water velocity in a wetted-wall column.

Calculation of Individual Coefficients

With interfacial and boundary conditions known, the individual coefficients can now be calculated. As the required equations are covered by Shepherd,[27] only a summary will be given.

The expressions for the average individual coefficients between horizontal planes in the tower are:

$$h = -\frac{Gs}{A}\int_{\text{Plane } N_1}^{\text{Plane } N_2} \frac{dt}{t_B - t_i} \quad ; \qquad (11)$$

$$k = -\frac{G}{A}\int_{\text{Plane } N_1}^{\text{Plane } N_2} \frac{dH}{H_B - H_i} \quad ; \qquad (12)$$

$$h' = -\frac{Gs}{A}\int_{\text{Plane } N_1}^{\text{Plane } N_2} \frac{dt}{t - t_i} \quad ; \qquad (13)$$

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[25] S. J. Friedman, C. O. Miller, Industrial and Engineering Chemistry 33, pp. 885-888, (1941).

[26] E. R. Gilliland and T. K. Sherwood, "Diffusion of Vapors into Air Streams," Industrial and Engineering Chemistry, 26 pp. 516-533, (1934).

[27] G. R. Shepherd, Ibid, pp. 11,12, (1947).

$$k' = -\frac{G}{A}\int_{\text{Plane } N_1}^{\text{Plane } N_2} \frac{dH}{H - H_i} \quad ; \qquad (14)$$

and

$$h_L = \frac{W}{A}\int_{\text{Plane } N_1}^{\text{Plane } N_2} \frac{dT}{T - t_i} \qquad (15)$$

The above integrals can be evaluated by the usual graphical methods. Individual coefficients at a particular plane in the tower can be determined by the corresponding differential form of the above equations.

DISCUSSION OF RESULTS

In determining interfacial conditions, a line of slope $\frac{h'}{k's}\frac{dH}{dt}$ through the point t,H usually intersected the equilibrium line below the water temperature at low negative values of $\frac{dH}{dt}$ when a value of 1.00 was assumed for $\frac{h'}{k's}$. However, as $\frac{dH}{dt}$ increased, the intersection usually occurred above the water temperature which, of course, is incorrect when both the air and water are cooling. Furthermore, if values appreciably lower than 1.00, 0,80 to 0.95, are assumed the point t_i, H_i appears to be too low at low negative values of $\frac{dH}{dt}$. In the latter case, the values for t_i, H_i are reasonable at higher values of $\frac{dH}{dt}$. There would, then, appear to be some question as to the validity of holding either $\frac{h}{ks}$ or $\frac{h'}{k's}$ constant over a wide range of humidities and temperatures. As previously mentioned, Shepherd[28] obtained a rather large difference between these ratios when $\frac{h}{ks}$ was assumed constant. When the interfacial conditions were determined by the intersection of a straight line through the points t,H and t", H" with the equilibrium line, a value for t_i lower than the water temperature was usually obtained at high negative values of $\frac{dH}{dt}$; however, at low negative values, the difference between the water temperature and the interfacial temperature was too low in some cases. This is particularly noticeable in the results of Run 1 which are shown in Table I of the appendix. It cannot be ascertained whether the correctness of the approximation actually diminished as the discrepancies were well within experimental error.

For the data shown in Tables I and II, the interfacial conditions were estimated by the intersection of a straight line through the point t, H and t", H"[29] in each case except for Run 7. In this latter case a value of 1.00 was assumed for $\frac{h'}{k's}$ in solving equation (5). The results in Table I show that a value of 0.95 would have been a better assumption as the difference between T and t_i at the various planes is very small. A satisfactory curve for t", H" could not be established for this run.

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[28] G. R. Shepherd, Ibid, p. 36, (1947).

[29] When the maximum temperature in the turbulent air core was not at the center of the column, the highest recorded value at a particular plane was assumed for t".

The available evidence is insufficient to determine whether interfacial conditions are estimated more accurately by assuming a constant value for $\frac{h}{ks}$, $\frac{h'}{k's}$ or $\frac{\alpha''}{\beta''}$ over a wide range of humidities and temperatures. However, the two techniques serve as a check and both are useful in evaluating the experimental data. A comparison of the two procedures is shown in Figures 4 and 7.

As previously mentioned, the boundary conditions t_B and H_B were obtained by estimating α' and β' from a plot of $\frac{t_y - t_i}{t'' - t_i}$ and $\frac{H_y - H_i}{H'' - H_i}$ vs. $\frac{y}{R_o}$ on logarithmic paper. Although the distance from the interface at which the slope of these curves became 1 varied widely, the values for both α' and β' appeared to be about the same. The data were not sufficiently accurate to prove or disprove the equality. Also, no trend could be established for a significant change in either α' or β' at the various planes, the variations being well within experimental error. For this reason the same value was used for both α' and β' during a particular run.[30] The actual values used ranged from 0.40 to 0.42 between runs. A considerable error in t_B and H_B may have resulted from this procedure. The relationship between the humidity ratio, $\frac{H_y - H_i}{H'' - H_i}$, and $\frac{y}{R_o}$ appears to be more accurate than the corresponding temperature relationship. The graphical determina-

[30]The change in α' and β' may be small as equations (3) and (4) show that the corresponding terms, α and β, do not vary greatly under the same flow conditions.

tion of α' and β' is shown in Figure 5.

The best estimate for the air film thickness from the various runs is about 0.03 inches; this compares with a value of approximately 0.034 reported by McAdams[31] for an air velocity of 13.1 feet per second. In each run, the air film thickness estimated by the temperature measurement is usually much smaller than the thickness indicated by the humidity measurement. Although both measurements are subject to a large error at distances near the interface, it is believed that the error in the temperature measurement is greater because of conduction along the thermocouple leads. Of course, small errors in placing the probes near the tower wall adversely affect the accuracy of both measurements.

The values obtained for $\frac{h'}{k's}$ are well within the range previously reported by other investigators. Average values of 1.01, 0.989, and 1.00 were found for $\frac{h}{ks}$, $\frac{h'}{k's}$ and $\frac{h''}{k''s}$, respectively. Furthermore, the variation in h' from 3.7 to 4.5 over the velocity range covered, 14 to 17 feet per second, is in good agreement with the respective values of 4.1 and 4.7 reported by McAdams[31] for the transfer of heat to air flowing in a tube. Values for the individual coefficients are shown in Table II.

Although probably not as accurate as h' and k', the values of h and k appear to be reasonable; however, a large

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[31] W. H. McAdams, Ibid, pp. 103, 165, (1942).

error exists in the value h_L. As the difference in temperature between the water and the interface is not great, small errors change this coefficient several fold. The average value of 35 for the ratio $\frac{h_L}{h'}$ shows that the main resistance to transfer is in the air film.

No attempt was made to correlate the limited amount of data collected; however, in comparing the results with the work of Shepherd, the over-all heat transfer coefficient, h', correlates roughly with the 0.8 power of the mass velocity.

In run 7, the flow of heat in the air stream reversed direction at point 13. From the plot of H vs. t shown in Figure 7, it is evident that the rate of change of the air temperature will be small as the humidity continues to increase after the reversal of heat flow.

Enthalpy balances[32] covering the surface area shown in Table I for the various runs ranged from -10.4 to +9.3 per cent. The error was usually greater at plane 15 or at planes 1 and 3 in the lower section of the column, and these points were omitted in some cases as shown in Table I. Variations in water temperature around the circumference of the tower at plane 15 could actually be detected by contacting the hand with the column. This condition, of course, affected the accuracy of both the humidity and water temperature measurements near the top of the column to a major extent.

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[32] No correction was made in any of the calculations for friction losses as the pressure drop through the tower was about 0.2 inches of water only.

The results for run 1 are not as accurate as the other runs reported. This was due to inexperience in operating the equipment, run 2 was omitted for this reason. In run 6, a leak was discovered in the humidity measuring equipment near the end of the run. This was particularly unfortunate because this run contained a temperature reversal point similar to run 7. Approximately twice the number of runs shown in Tables I and II were started but not completed because of mechanical failures in the equipment or changes in atmospheric conditions which significantly affected equilibrium in the tower. No number was given to such incompleted runs.

CONCLUSIONS

The results obtained for the several coefficients in the air stream at velocities ranging from about 14 to 17 feet per second are believed to be valid. This conclusion is supported by the agreement of both h' and the ratio $\frac{h'}{k's}$ with previously observed values. No attempt was made to correlate the limited amount of data; however, in comparing the results with the data of Shepherd, the over-all heat transfer coefficient correlates roughly with the 0.8 power of the mass velocity. The error in h_L is large, and very accurate data would be required to determine the correct value of this coefficient by the experimental procedure used in this investigation.

In determining interfacial conditions, caution must be used in assuming $\frac{h}{ks}$, the corresponding ratio $\frac{h'}{k's}$, or the ratio $\frac{\alpha''}{\beta''}$ constant over a wide range of temperatures and humidities. The available evidence does not permit a firm comparison of the procedures; however, the results obtained in this work indicate that both methods are a good approximation.

The determination of point to point conditions within a wetted-wall tower is of considerable aid in visualizing the mechanism of heat and mass transfer. In future work, the velocity distribution should also be established to permit a direct comparison between the transfer of heat, mass, and momentum.

As a considerable error was introduced in the manual regulation of flow rates and temperatures, precision control equipment should be provided in future studies along this line; the automatic control of the plate temperature in the humidity apparatus should also be included. The temperature measuring equipment is adequate, as such; however, an improvement is needed in design to facilitate the accurate placement of the probe at points on both sides of the center of the column, particularly near the tower wall.

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APPENDIX I

TABLE I

Point Conditions Within Tower

Pt in Tower	T	t	t_B	t_i	H	H_B	H_i	G	W	A
Run 1										
8	78.7	100.1	88.1	78.7	0.0109	0.0171	0.0221	118	82.5	1.96
10	79.4	98.1	87.7	79.1	0.0116	0.0175	0.0224			
12	80.2	96.4	87.3	79.8	0.0125	0.0182	0.0229			
14	81.4	95.1	87.1	80.4	0.0136	0.0190	0.0234			
Run 3										
3	78.7	103.8	88.3	76.4	0.0071	0.0150	0.0204	127	49.5	3.58
6	81.6	102.4	88.6	79.1	0.0083	0.0166	0.0224			
8	83.7	100.7	88.7	79.7	0.0092	0.0169	0.0228			
10	86.1	98.9	88.8	80.4	0.0102	0.0172	0.0233			
12	88.8	97.6	89.4	82.7	0.0116	0.0190	0.0252			
14	92.0	96.9	90.9	85.9	0.0134	0.0214	0.0281			
Run 4										
5	81.5	102.6	89.2	78.9	0.0083	0.0162	0.0223	137	49.5	3.26
7	83.5	100.7	89.5	80.2	0.0089	0.0168	0.0234			
9	86.0	99.1	90.2	82.9	0.0098	0.0182	0.0255			
11	88.7	97.8	91.7	86.3	0.0112	0.0204	0.0286			
13	91.4	97.2	93.9	90.9	0.0128	0.0237	0.0334			
15	94.3	97.0	94.8	92.6	0.0150	0.0249	0.0352			

APPENDIX I

TABLE I (Cont'd)

Point Conditions Within Tower

Pt in Tower	T	t	t_B	t_i	H	H_B	H_i	G	W	A
Run 5										
1	78.4	105.1	87.3	74.0	0.0077	0.0141	0.0189	144	49.5	3.91
3	80.2	102.9	89.1	78.9	0.0087	0.0165	0.0223			
5	82.0	101.0	90.1	81.7	0.0098	0.0181	0.0246			
7	84.1	99.5	91.1	84.5	0.0109	0.0199	0.0269			
9	86.8	98.4	91.8	86.4	0.0122	0.0212	0.0287			
11	90.2	97.6	92.8	88.7	0.0137	0.0230	0.0310			
13	94.2	97.0	93.9	91.3	0.0157	0.0253	0.0337			
Run 7										
1	81.1	103.2	88.0	76.2	0.0099	0.0157	0.0202	144	49.5	3.26[33]
3	83.3	100.9	90.2	81.9	0.0109	0.0186	0.0244			
5	85.6	99.9	92.2	86.6	0.0120	0.0215	0.0287			
7	88.2	99.2	93.6	88.8	0.0131	0.0230	0.0308			
9	91.2	98.6	94.7	91.6	0.0143	0.0252	0.0338			
11	94.6	98.2	96.2	94.7	0.0157	0.0278	0.0374			
13	98.2	98.1	98.1	98.1	0.0173	0.0309	0.0417			
15	102.1	98.2	99.0	99.6	0.0194	0.0322	0.0438			

[33] The area, A, is shown for points 1 through 11 only because the heat flow reverses direction at point 13.

APPENDIX I

Table II. Calculated Values of Individual Coefficients.

Run no.	Air velocity u_{ave} (ft./sec.)	Relative[34] velocity air to water (ft./sec.)	h	k	$\frac{h}{ks}$	h'	k'	$\frac{h'}{k's}$	$\frac{h''}{k''s}$	h_L	$\frac{h_L}{h'}$
1	14.1	15.3	9.05	35.0	1.06	4.09	15.8	1.06	1.07	280	68
3	15.2	16.4	8.70	37.1	0.956	3.73	16.3	0.935	0.952	47	13
4	16.4	17.6	8.36	33.6	1.02	3.86	16.0	0.984	0.985	80	21
5	17.3	18.5	9.95	42.1	0.966	4.48	18.8	0.977	1.01	216	48
7	17.2	18.4	9.16	35.6	1.04	4.00	15.5	1.04	--	77	19

[34] The relative velocity refers to the average velocity of the air stream and the surface velocity of the water film.

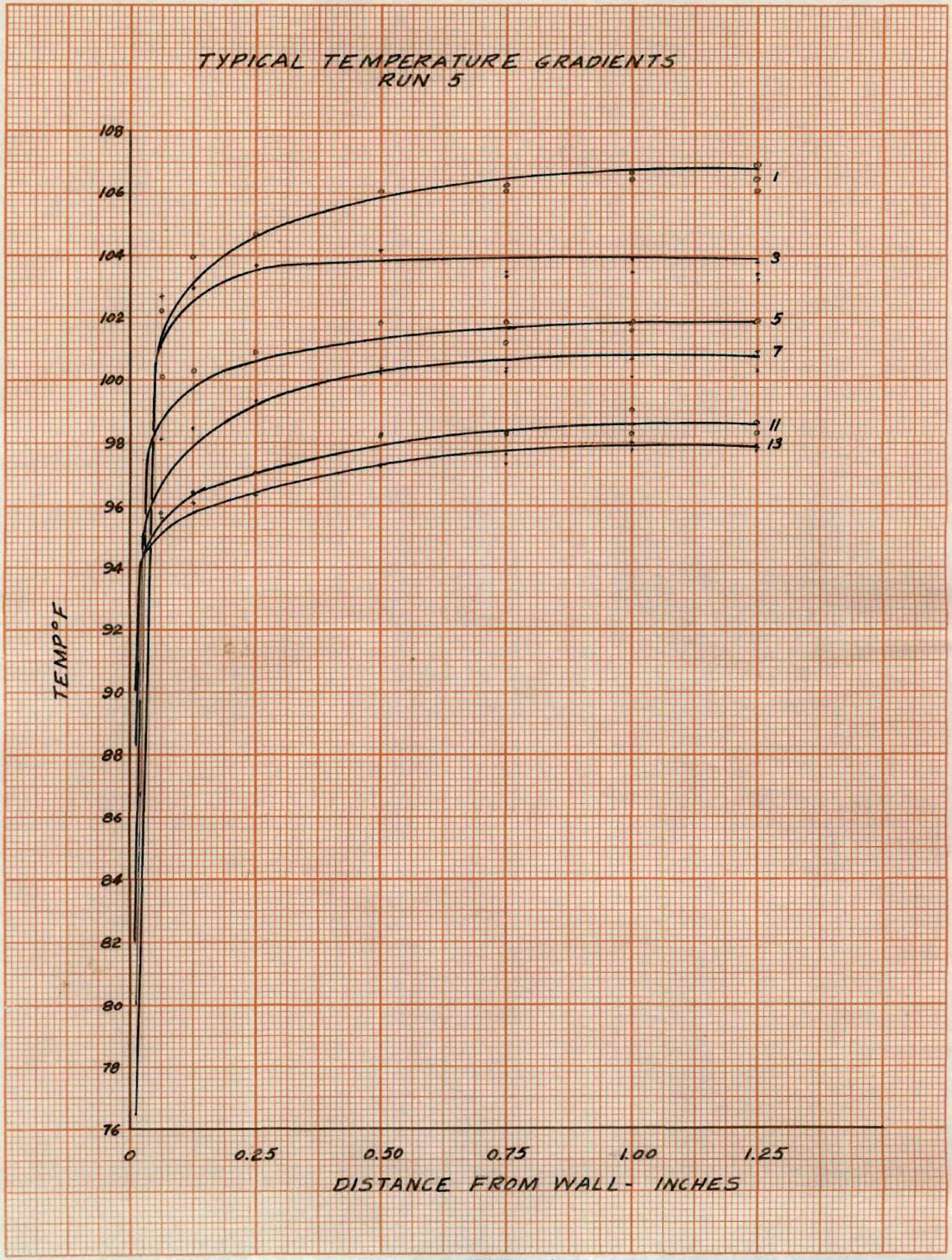

FIG 1

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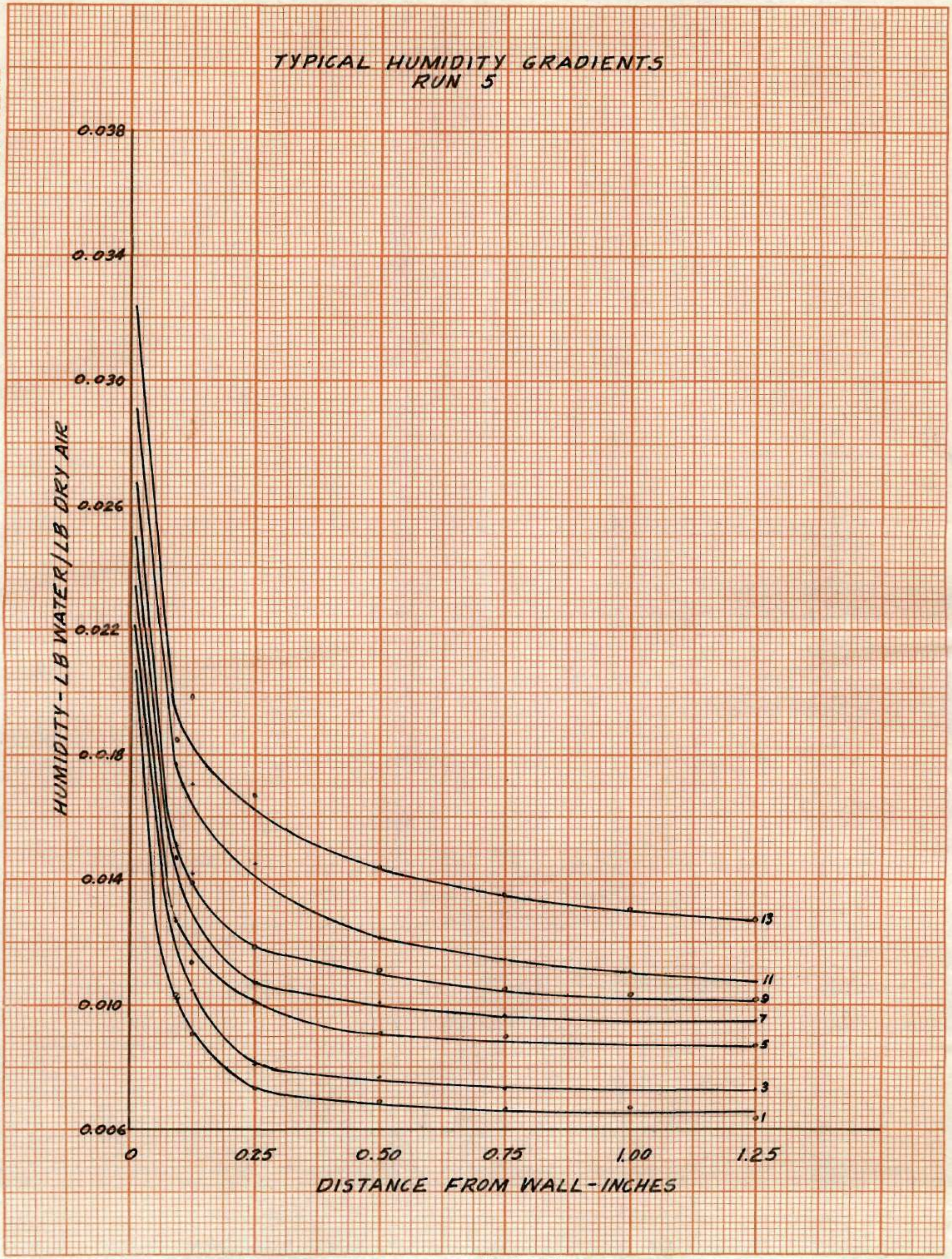

FIG 2

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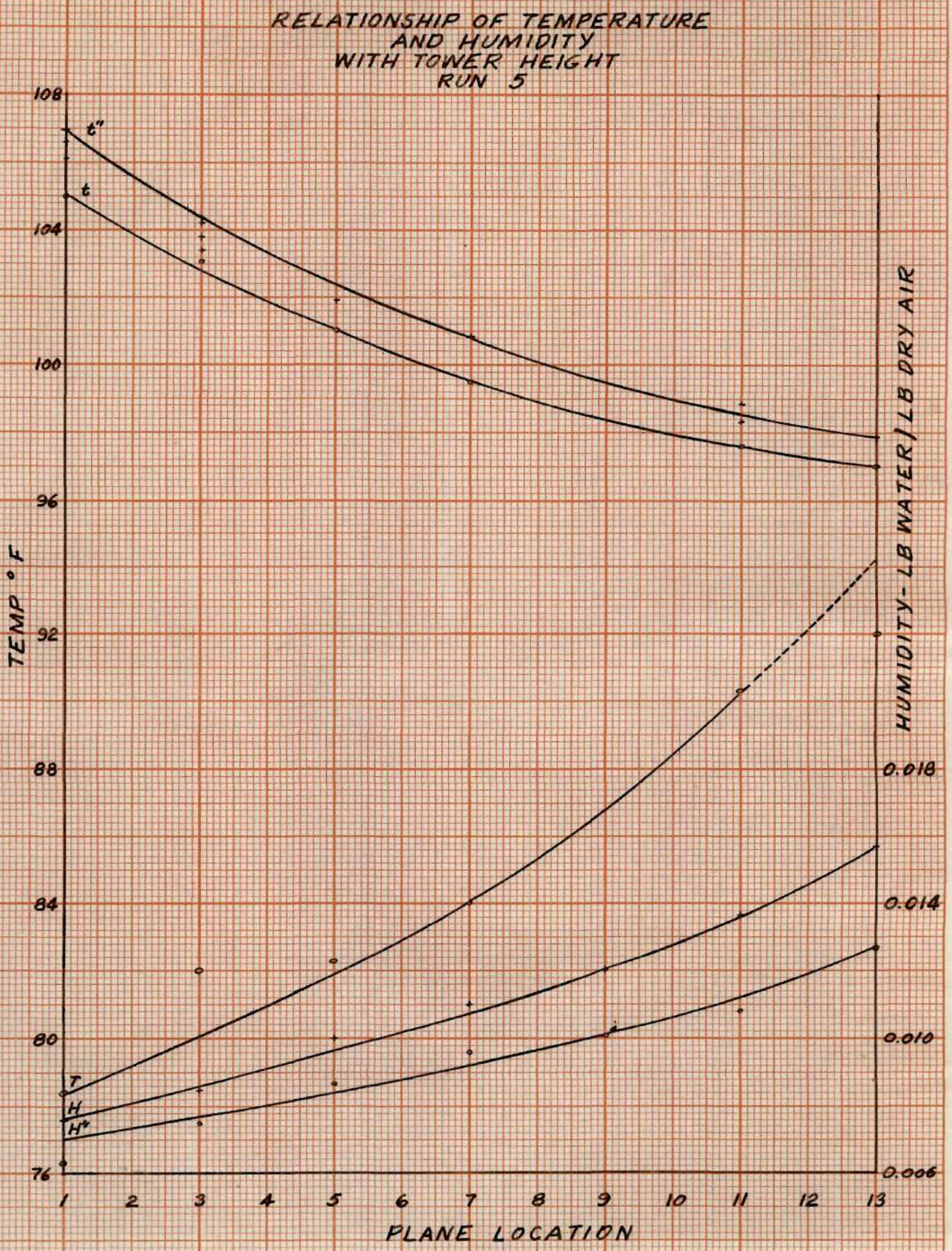

FIG 3

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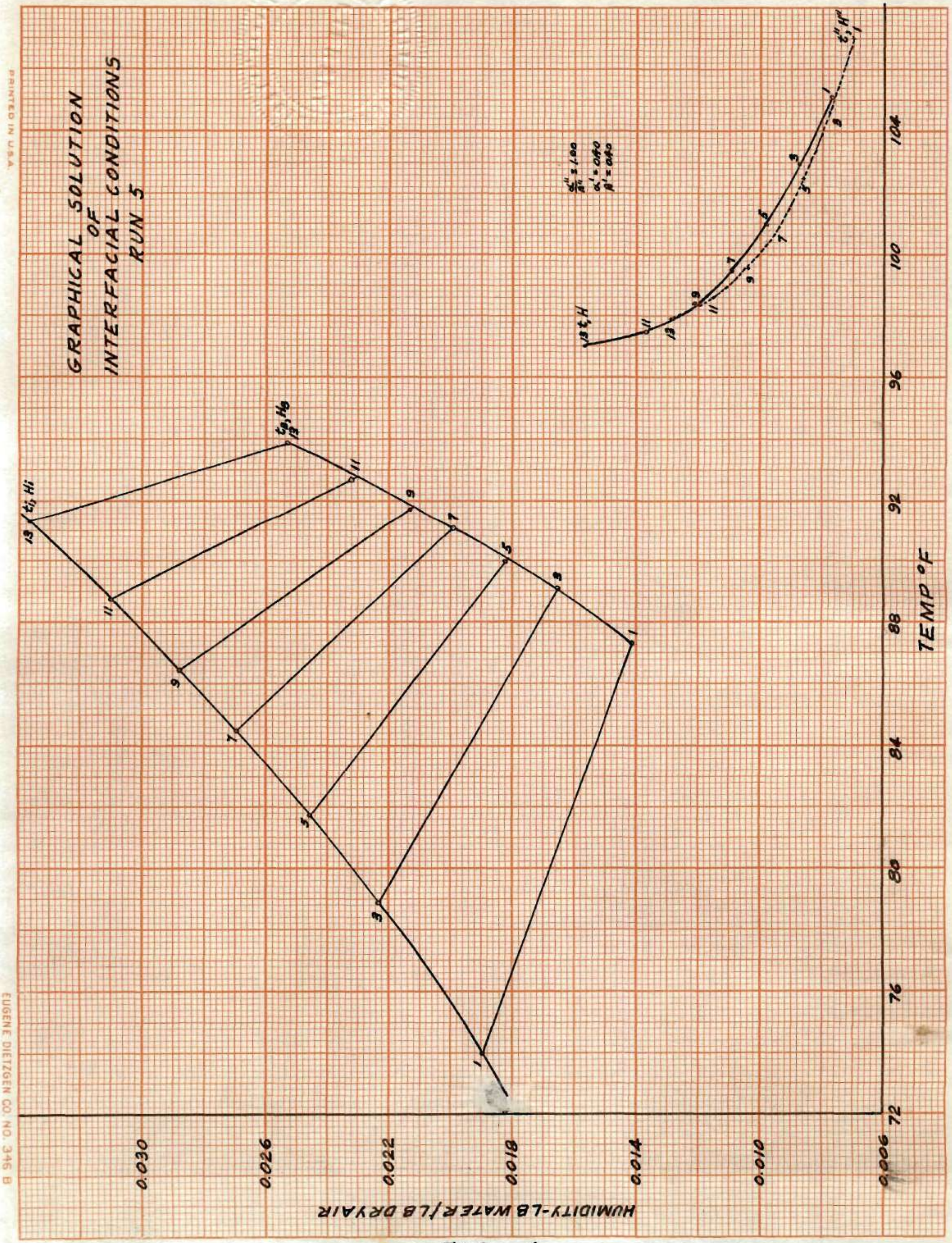
GRAPHICAL SOLUTION
OF
INTERFACIAL CONDITIONS
RUN 5
α''/β'' = 1.00
α' = 0.40
β' = 0.40
t_i, H_i
t_G, H_G
t, H
t_i'', H''
TEMP °F
HUMIDITY-LB WATER/LB DRY AIR
72
76
80
84
88
92
96
100
104
0.030
0.026
0.022
0.018
0.014
0.010
0.006
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FIG 4

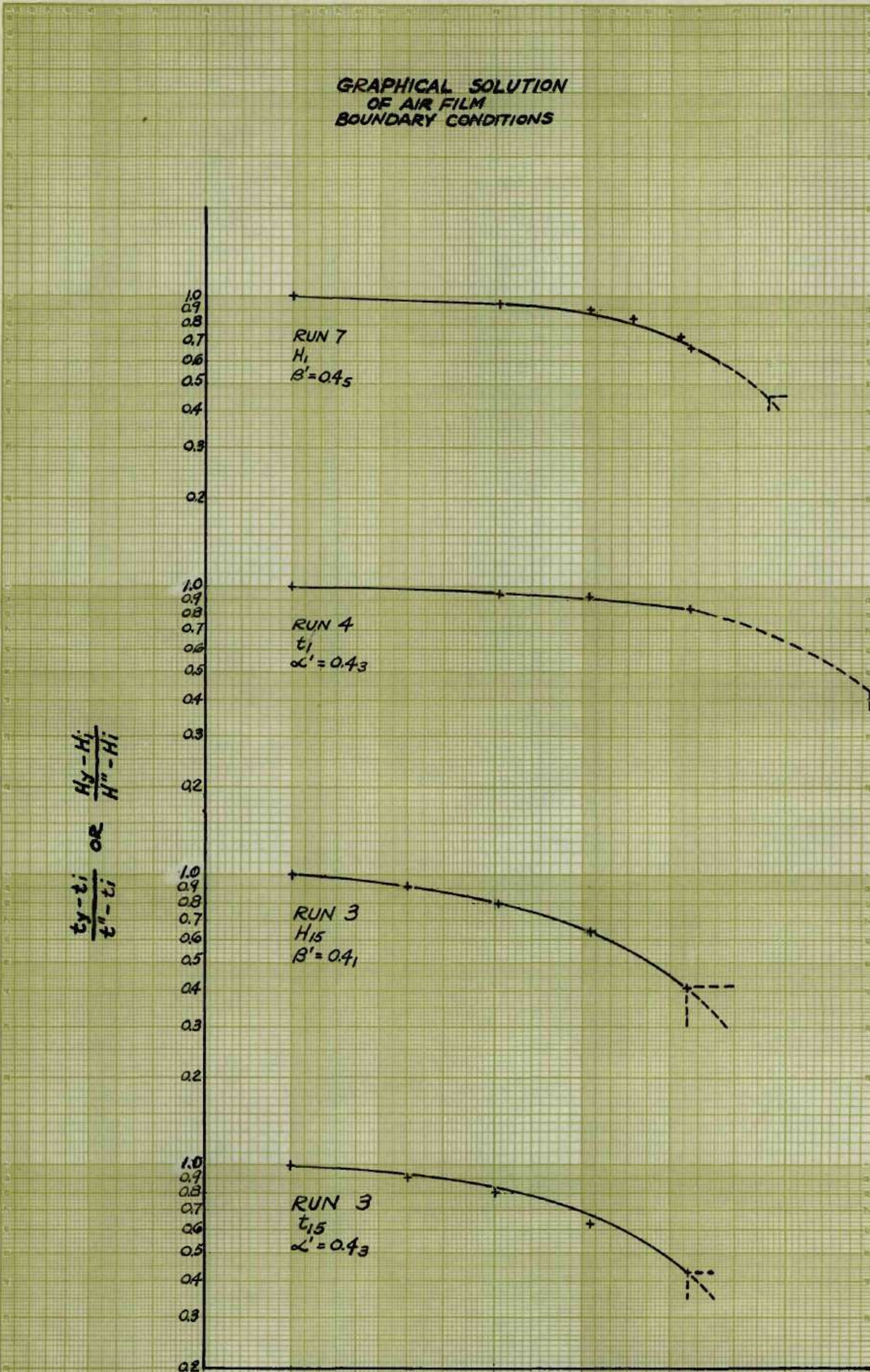
GRAPHICAL SOLUTION
OF AIR FILM
BOUNDARY CONDITIONS
RUN 7
H1
β'=0.45
RUN 4
t1
α'=0.43
RUN 3
H15
β'=0.41
RUN 3
t15
α'=0.43
ty-ti / t"-ti OR Hy-Hi / H"-Hi
y / Ro

FIG 5

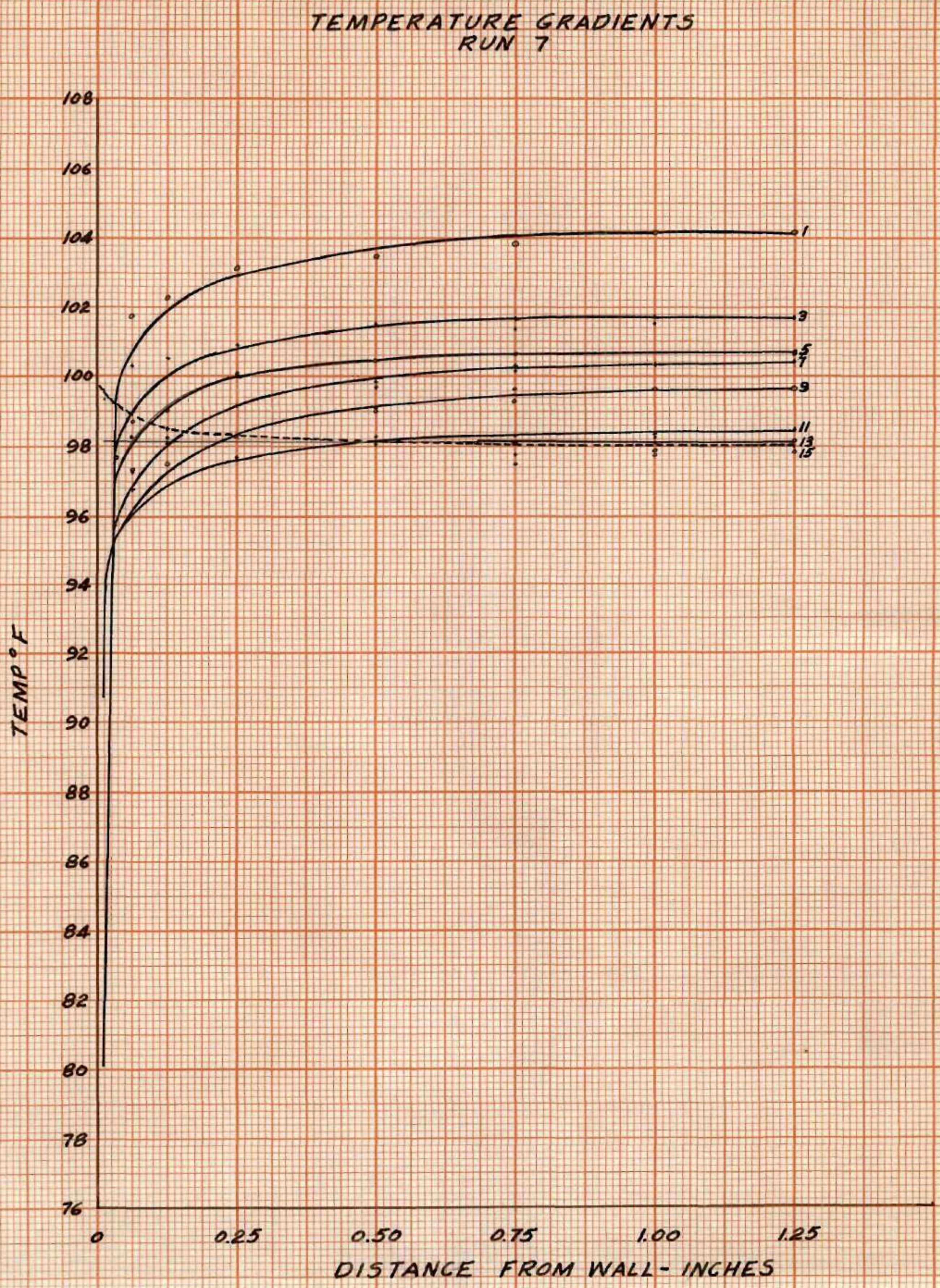

FIG 6

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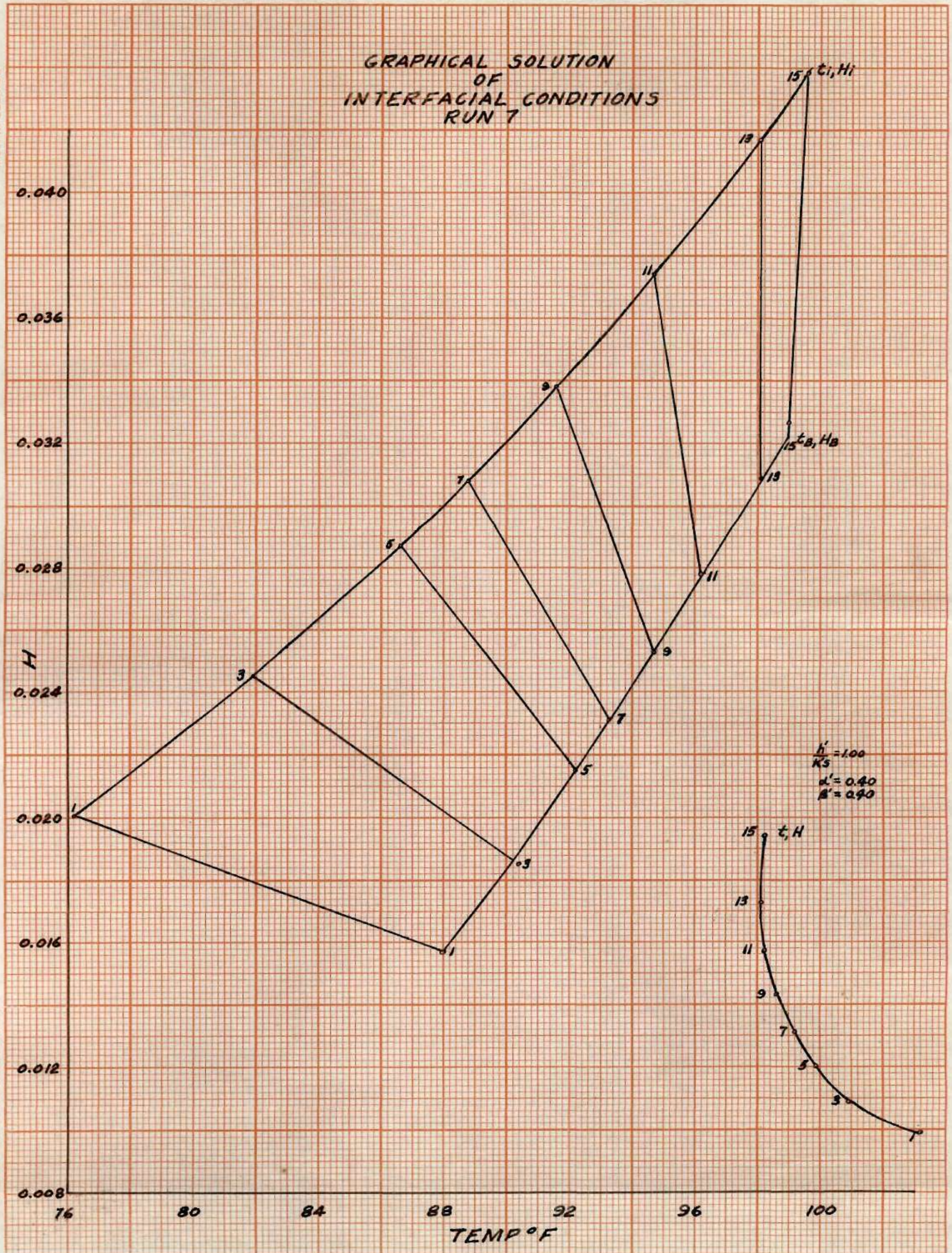

FIG 7

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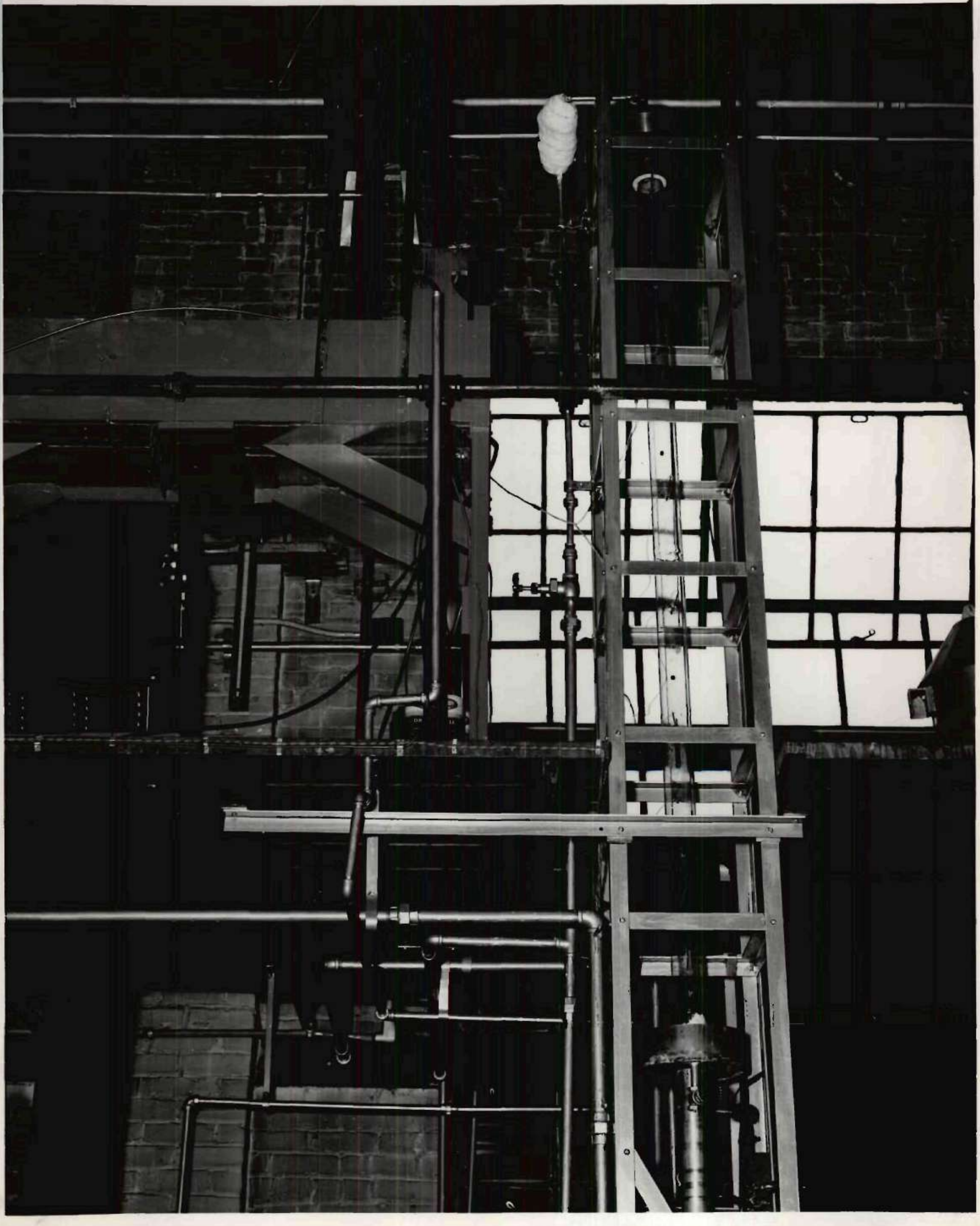

Figure 8
Tower Installation

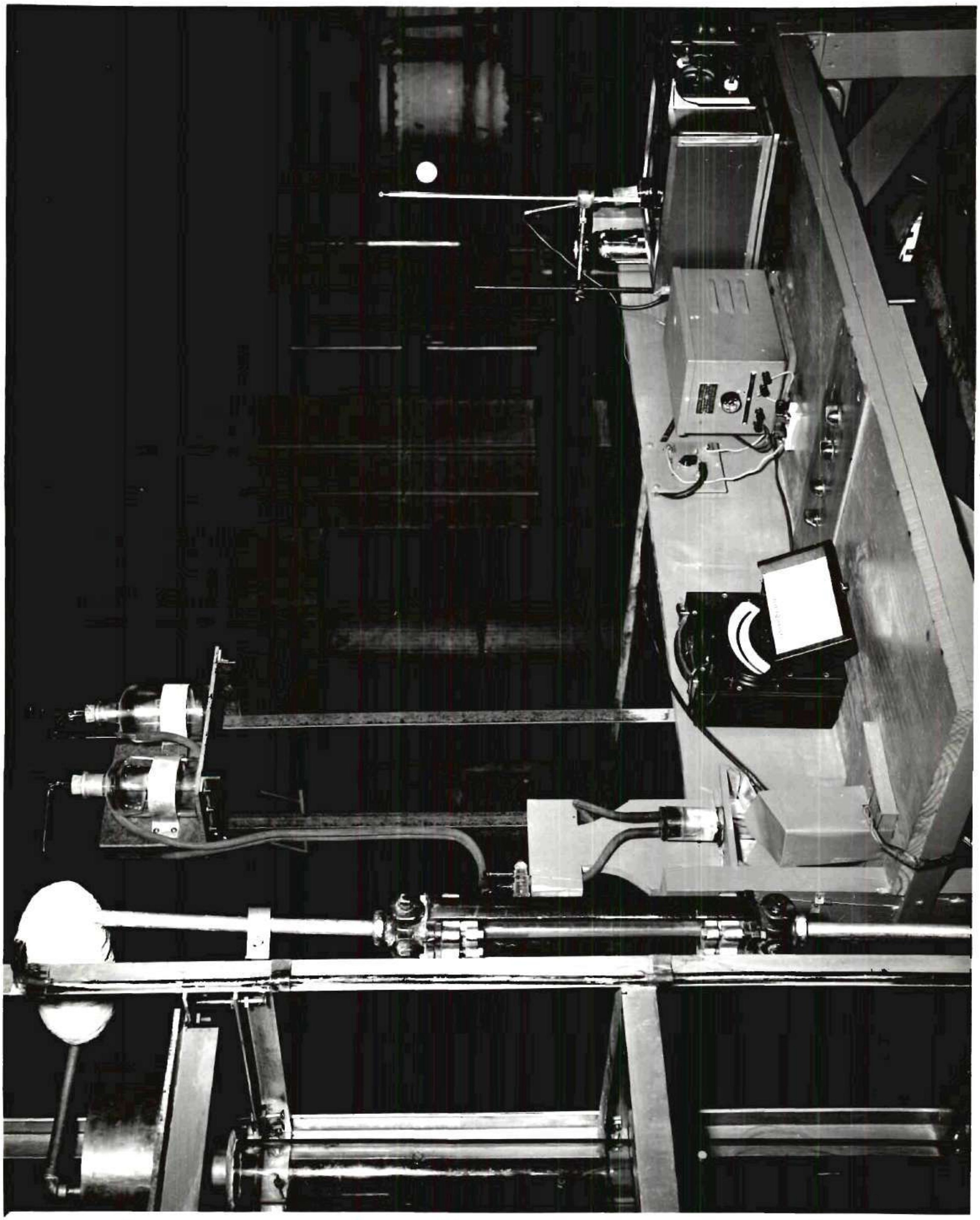

Figure 9
Humidity and Temperature Measuring Equipment

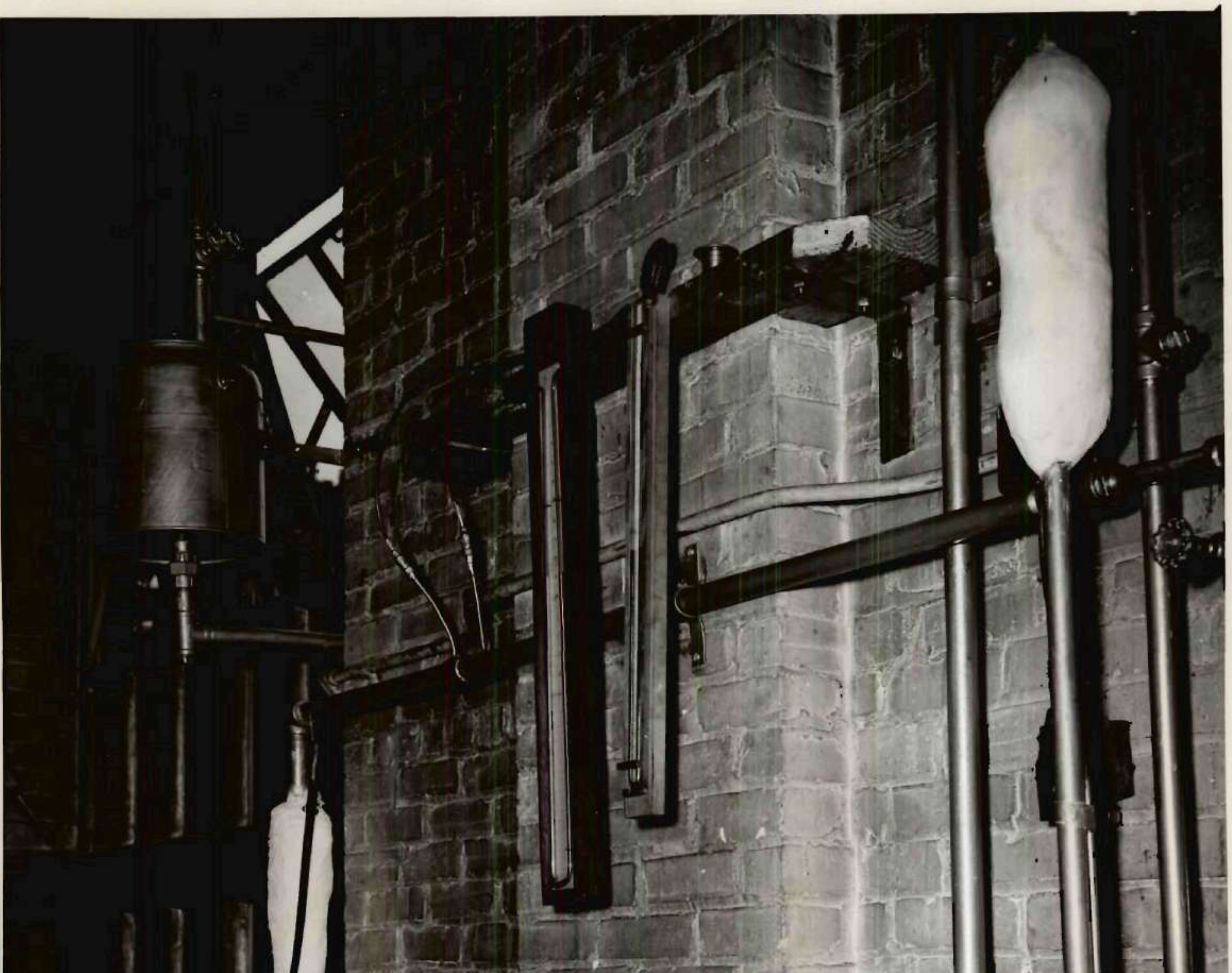

Figure 10
Air Flow and Temperature Regulating Equipment